Lec 27: 11/27/2013

Cosmic Microwave Background (Cont'd):

Silk Damping:

So far, we have assumed a perfect fluid consisting of baryons and photons with a tight coupling between the two. In this regime, baryons drag photons with 100% efficiency, and this is the reason why baryon perturbations oscillate and temperature fluctuations follow them closely. The tight coupling regime, however, breaks down at very small length scales (or very high multipoles):

(1) Photon mean free path: The pressure in the baryon-photon fluid is provided by Thomson scattering of photons off electrons, which is then transmitted to baryons by the Coulomb interactions. However, the photons move freely between two successive scatterings. The mean-free-path, denoted by λ_γ, is given by:

$$\lambda_\gamma = \Gamma_T^{-1} = (n_e \sigma_T)^{-1}$$

It is obvious that free motion of photons at length scales below λ_γ erases any fluctuation in n_γ. One therefore does not expect to see any temperature fluctuations in the CMB photons at multipoles that correspond to wavelengths below λ_γ. The comoving length corresponding to the mean-free-path of photons is:

$$(\lambda_\gamma)_{com} \sim 2.5 \text{ Mpc} \qquad (I)$$

At the time of recombination this corresponds to a very high multipole $\ell \sim O(10000)$. We recall that the horizon radius at the time of recombination corresponds to a multipole $\ell \sim O(100)$.

(2) Photon diffusion length: The photons basically undergo a random walk due to collisions with electrons. Diffusion also washes out fluctuations in n_γ, and hence temperature fluctuations

The question is whether diffusion is more important for a perturbation of given wavelength or acoustic oscillations. It is clear that perturbations whose wavelength is smaller than the diffusion length of photons, denoted by λ_D, will be significantly affected. This is the so calle "Silk damping" (or Collisional damping). The distance travelled by a random walk increases $\propto \sqrt{t}$. This results in the following relation:

$$\lambda_D \sim \sqrt{H^{-1} \lambda_\gamma}$$

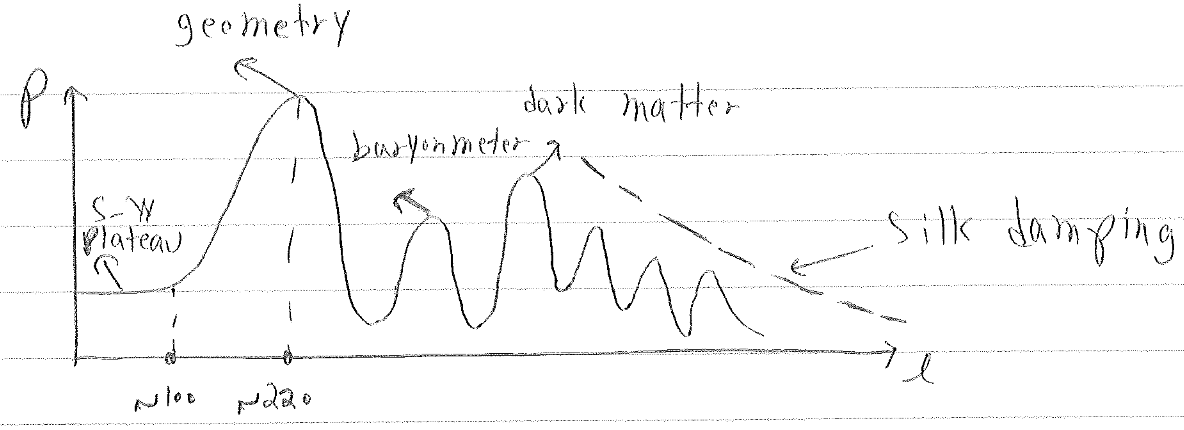
Translating this into a delation among multipoles, we have:

$$\ell_D \sim \sqrt{\ell_H \, \ell_\gamma}$$

At the time of recombination, this yields:

$$\ell_D \sim O(1000) \qquad (\text{II})$$

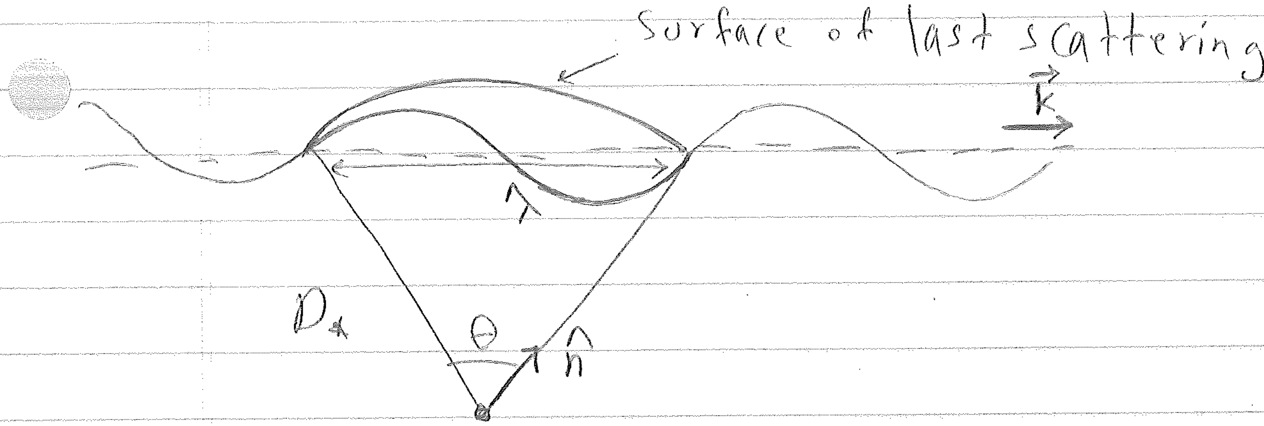
Therefore, all modes beyond the 3rd acoustic peak are affected by Silk damping, which results in their suppression:



One comment is in order. So far, we have mainly discussed the evolution of perturbations with a given wavelength. However, the CMB provides a snapshot of many modes at the time of recombination. The oscillatory pattern in the CMB power spectrum (over k or l) is directly related to that for a given mode over t. Modes with a higher l have a shorter wavelength. They therefore entered the horizon at an earlier time and had a longer time to evolve. The first acoustic peak then corresponds to the mode that had maximum compression at t_{rec}, while the 2nd peak corresponds to the mode at its rarefaction, etc. Modes in between were at different points between compression and rarefaction, which is the reason why we see an oscillatory pattern.

Spatial vs Angular Power:

So far, we have focused on spatial modes of perturbations obtained via Fourier decomposition, and related them to multipoles according to $\lambda = \frac{2\pi}{k} \sim D_* \theta \sim D_* \frac{180^\circ}{\ell}$, where D_* is the comoving distance between us and the surface of last scattering:



This is intuitively understandable. Here we present a concrete derivation of this correspondence.

$$\frac{\delta T}{T}(\hat{n}) = \sum_{\ell, m} T_{\ell m} Y_{\ell m}(\hat{n}) \qquad \frac{\delta T}{T}(\vec{x}) = \int \frac{d^3 k}{(2\pi)^3} \frac{\delta T}{T}(\vec{k})\, e^{i \vec{k}\cdot\vec{x}}$$

$$\langle T_{\ell m} T_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$$

↳ statistically isotropic fluctuations

$$\left\langle \frac{\delta T}{T}(\vec{x}) \frac{\delta T}{T}(\vec{x}') \right\rangle = \int \frac{d^3 k}{(2\pi)^3} P_T(k) = \int d\ln k\, k^3 \frac{P_T(k)}{2\pi^2} = \int d\ln k\, \Delta_T^2(k)$$

$$\left\langle \frac{\delta T}{T}(\vec{k}) \frac{\delta T}{T}(\vec{k}') \right\rangle = (2\pi)^3 \delta(\vec{k}-\vec{k}')\, P_T(\vec{k})$$

→ power spectrum

$$\frac{\delta T}{T}(\hat{n}) = \int \frac{d^3k}{(2\pi)^3} \frac{\delta T}{T}(\vec{k})\, e^{i\vec{k}\cdot D_* \hat{n}}$$

$$e^{i\vec{k}\cdot D_*\hat{n}} = 4\pi \sum_{l,m} i^l j_l(kD_*)\, Y^*_{lm}(\vec{k})\, Y_{lm}(\hat{n})$$

↑ partial wave expansion ↘ $k \equiv |\vec{k}|$

$$j_l(\rho) = (-\rho)^l \left(\frac{1}{\rho}\frac{d}{d\rho}\right)^l \left(\frac{\sin\rho}{\rho}\right) \quad \text{(Spherical Bessel functions)}$$

$$T_{lm} = \int \frac{d^3k}{(2\pi)^3} \frac{\delta T}{T}(\vec{k})\, 4\pi i^l j_l(kD_*)\, Y_{lm}(\vec{k}) \Longrightarrow \langle T^*_{lm} T_{l'm'} \rangle$$

↓ orthonormality of spherical harmonics

$$\int \frac{d^3k}{(2\pi)^3} (4\pi)^2 i^{l-l'} j_l(kD_*)\, j_{l'}(kD_*)\, Y^*_{lm}(\vec{k})\, Y_{l'm'}(\vec{k})\, P_T(k)$$

$$= \delta_{ll'} \delta_{mm'}\, 4\pi \int d\ln k\; j^2_l(kD_*)\, \Delta^2_T(k)$$

For a slowly varying $\Delta^2_T(k)$, which is what we observe, we have:

$$\langle T^*_{lm} T_{l'm'} \rangle \approx \delta_{ll'} \delta_{mm'}\, 4\pi \Delta^2_T(k) \underbrace{\int d\ln k\; j^2_l(kD_*)}_{\frac{1}{2l(l+1)}} \Rightarrow$$

$$C_l = \frac{2\pi \Delta^2_T\left(\frac{l}{D_*}\right)}{l(l+1)}$$

The commonly used logarithmic power is given by:

$$\Delta^2_T\left(\frac{l}{D_*}\right) = \frac{l(l+1)\, C_l}{2\pi} \qquad \text{III}$$

↗ angular mode (l)

↳ spatial mode ($\frac{l}{D_*}$)